MATHEMATICAL MODELS OF A REQUIRED POWER DESIGN FOR IRRIGATION WITH SMALL-SIZED ELECTRIC MOTORS[i]

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ABSTRACT

Irrigation methods in rural areas usually consist of pumping water from dams and reservoirs by engines connected to the electric power station or powered with diesel oil. Current assay establishes mathematical models for scaling low-power electric motors for irrigation in small cultivated areas. Equations of engine power were determined by numerical integration and by the theorem of kinetic energy. A geometric model was established with a paraboloid of revolution to determine the volume of the reservoir. A 3600 m² area was irrigated during 4 h, 380 m distant from the dam, with a 21° slope, for simulation purposes. The amount of water for irrigation was 5 U m² by means of a dam with diameter 14 m and 5 m deep. The establishment of mathematical models scaled up a 2.5 hp engine with a removal of 5000 L of water from a 385 000 L dam, with a variation of 7.2 cm and the immersion of the engine below this borderline.

Keywords: water pumping, numerical integration, kinetic energy.

RESUMO

Modelos matemáticos de dimensionamento da potência necessária para a irrigação com motores elétricos de pequeno porte. É comum em áreas rurais a utilização de métodos de irrigação que consistem no bombeamento de água de represas e açudes, feitos em geral por motores conectados à rede elétrica ou utilizando óleo diesel. Este trabalho visa determinar modelos matemáticos para dimensionamento de motores elétricos de baixa potência utilizados para irrigação de pequenas áreas plantadas. A determinação das equações da potência do motor foi realizada por integração numérica e pelo teorema da Energia Cinética. Para a determinação do volume da represa, foi feita sua modelagem geométrica com um parabolóide de revolução. A obtenção dos resultados práticos considerou, para efeito de simulação, uma área de 3600 m² a ser irrigada em 4 h, distante de 380 m da represa a um terreno com inclinação 21º. A quantidade de água para irrigação foi de 5 L m^{-2}, por meio de uma represa de diâmetro 14 m e profundidade 5 m. Com os modelos matemáticos criados, dimensionou-se

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um motor de 2,5 cv e que seriam retirados 5000 L de água da represa de volume 385.000 L, havendo uma variação no nível de 7,2 cm, sendo necessária imersão do motor abaixo deste limítrofe.

Palavras-chave: bombeamento de água, integração numérica, energia cinética.

INTRODUCTION

Irrigation is an agricultural practice for the supply of water to plants in places and at times when rainfall or other natural supply sources are insufficient to provide for their water requirements (SILVA; KLAR, 2010).

According to Fedrizzi (1997), the pumping of water for human consumption, domestic animals and irrigation provides drinkable water to populations, the maintenance of herds and the increase in agricultural production and productivity and the seasonal regularity of production with food throughout the year and not merely in the rainy season.

The dimensioning of water pumping devices is an important stage for the performance of irrigation pressurized systems (ZOCOLER et al., 2013). Although concern is primarily focused on the size of the hydraulic apparatus, the feasibility of the enterprise depends on the efficiency and costs of its installation and functioning (CARVALHO; OLIVEIRA, 2008).

In general, the utilization of mathematical modeling in agrarian sciences, among many applications, aims at realizing estimates (PEREIRA et al., 2008), approach of qualitative variables and correction of errors using numerical methods (GABRIEL FILHO et al., 2011a, 2011b).

The correct sizing of an irrigation system requires the determination of maximum water requirements demanded by the plant (SANTOS JÚNIOR et al., 2014), the necessary time for irrigation (DADHICH et al., 2012) and the maximum available discharge of the water course without the jeopardizing of the latter (KUSTU et al., 2010) for the adequate use of the pumping system.

Suction systems for the pumping up of water from dams, lakes or rivers are a highly common practice for the irrigation of plants in rural areas. The system works by pressure and water is pumped to where a certain type of culture is being cultivated (Figure 1).

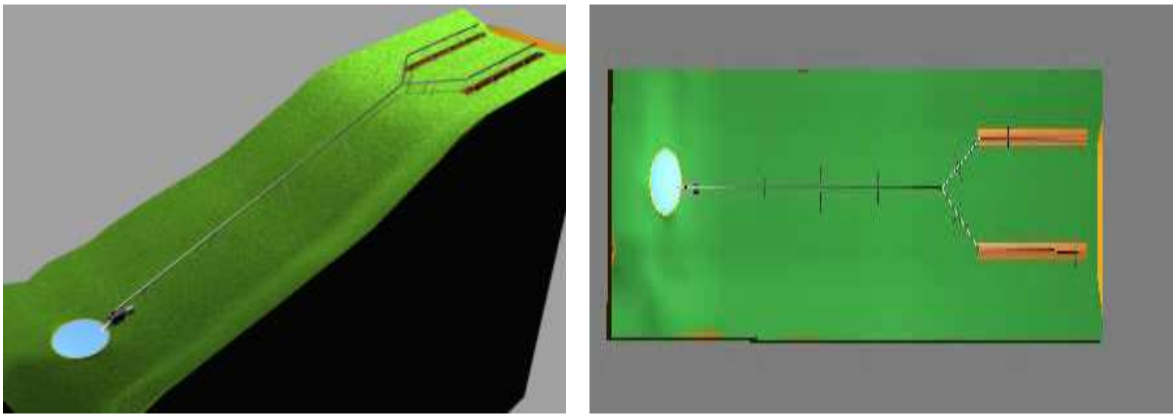

Figure 1 – Cross-section of an irrigation system.

As a rule, electric energy or diesel oil run engines do the pumping. In small cultivated areas, such as in the plantations of vegetables, low-power electric engines are employed due to their cost savings and good performance

Water pumping systems with renewable types of energy supply have been analyzed by Kolling et al. (2004) and

Michels et al. (2009), who have also assessed different conditions of discharge and determined the daily pumping of the evaluated apparatuses.

The precise dimensioning of engines for water pumping may be found in the research by Oliveira Filho et al. (2010) who analyzed the potency and the super-sizing of engines with regard to the unit for the functioning of the hydraulic pump.

Consequently, when the irrigation system is implemented, the correct dimension is mandatory due to the high efficiency of the system (LANKFORD, 2012). Models for the dimensioning of systems and the management of the resources are very important to reduce excessive costs (DECHMI et al., 2012; SAMMIS et al., 2012). However, technological development for the automatization of irrigation is highly feasible for the decrease of applied water volume (ROMERO et al., 2012; KAMASH, 2012).

When the system´s dimensioning is not performed correctly, relevant regional damage may occur, as reported by Borgia et al. (2012) for Mauritania and Spain, and by Skhiri and Dechmi (2012) on the leeching of nutrients by excess of water due to irrigation with low efficiency and bad dimensioning. A similar factor has been reported in China where decrease in culture productivity has been occurring in certain regions (JIANG et al., 2012).

Current assay determines the mathematical models for the dimensioning of low-power electric engines for the irrigation of small cultivated areas, through simulations in the pump´s power variations, water volume in the reservoir, variation in water level and other factors within a specific mathematical modeling.

MATERIALS AND METHODS

For simulation purposes, practical results focused on an area of 0.36 hectares, irrigated for 4 hours, at a distance of 380 m from the dam, on land with a 21º slope. Water applied to the cultivated area amounted to 5 $mm.day^{-1}$, with a 2 $L.s^{-1}$ discharge recommended to avoid cavitation, from a reservoir measuring 14 m diameter and 15 m depth.

A geometric model with a paraboloid of revolution was constructed to determine the dam´s volume. Let us consider a dam full of water, with a circle-shaped surface and a certain depth. The first hypothesis investigates whether the volume of water in the dam is sufficient for the required irrigation according to conditions of the proposed situation. A Cartesian plane was prepared in which the axis of the abscissae passes on the water surface and the axis of the ordinates passes through the center of the dam, coupled to a parabola in which the sides pass approximately through the side of the dam (Figure 2).

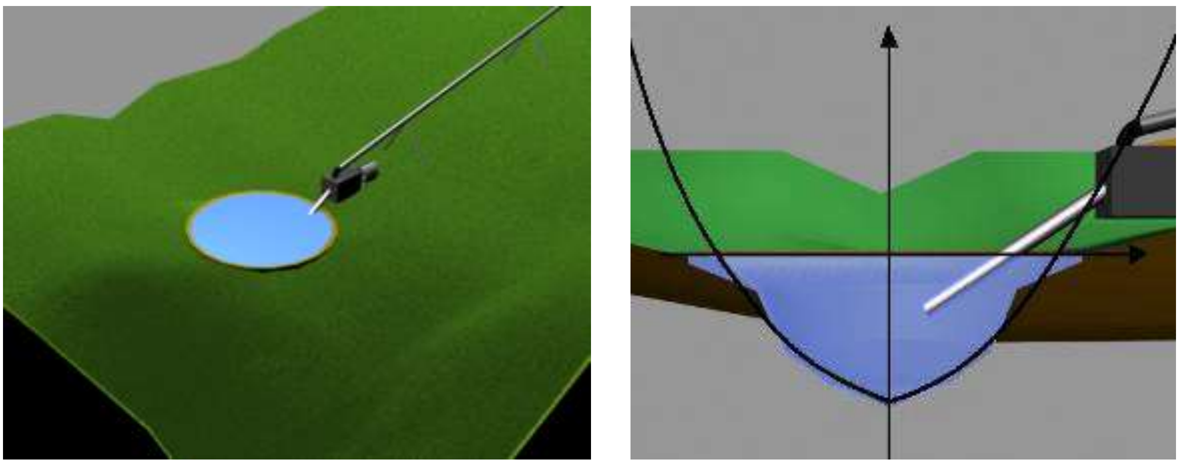

Figure 2 – Approximate parabola of the dam.

The point corresponding to the depth of the dam shaped $(0, -p)$ and the lateral points corresponding to the margin, namely $\left(-{}^{d}/_{2}, 0\right)$ and $\left({}^{d}/_{2}, 0\right)$, in which d is the diameter, should be found to determine the function $g(y)$ that describes the parabola´s right branch (Figure 2). Further, the coordinates of the points in the quadratic expression $(x) = ax^2 + bx + c$ and determine the rates of a, b and c should be substituted. Since $0 \leq x \leq {}^{d}/_{2}$ and $y = f(x)$, $x = f^{-1}(y)$ is obtained, coupled to the function $g(y)$ which, following Larson (2011), is given by $g(y) = f^{-1}(y)$.

The volume of the solid provided by the rotation of the right branch of the parabola around the axis of the ordinates (Figure 3) is approximately equivalent to the volume of the water in the dam.

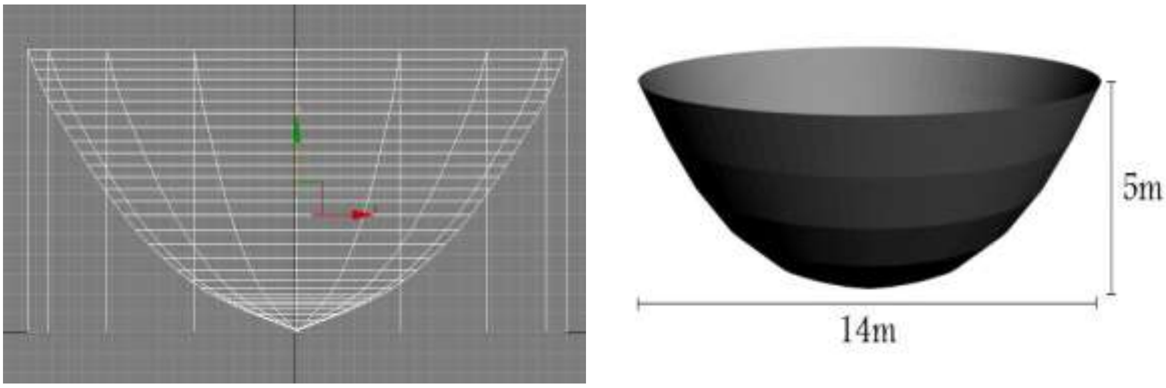

Figure 3 – Solid obtained by the rotation of the parabola around axis y.

According to Stewart (2008), the volume of the solid may be calculated as follows:

$$V = \lim_{|\Delta| \to 0} \sum_{i=1}^{n} \pi [g(y)]^2 \Delta_{iy} = \pi \int_{a}^{b} [g(\mathrm{y})]^2 \, \mathrm{dy} \quad (1)$$

Formula (1) should also be employed to calculate the variation of the water level when the pumped water volume is previously informed.

The equations of electric power were determined by the theorem of Kinetic Energy. According to Calçada and Sampaio (1998), the application of the theorem to a certain portion of water with mass m and volume V leads provides the following expression:

$$T_R = \frac{mv^2}{2} - \frac{mv_0^2}{2} \quad (2)$$

If P_X is the weight of the portion of water, θ is the slope angle and F is the force that the pump exerts on the portion of water, then:

$$T_{P_X} + T_F = \frac{m \cdot v^2}{2} - \frac{m \cdot v_0^2}{2} \Rightarrow -m \cdot g \cdot sen\theta \cdot \Delta s + T_F = \frac{m \cdot v^2}{2} - \frac{m \cdot {v_0}^2}{2}$$

If $m = d.V$ and $v_0 = 0$, we have:

$$-d \cdot V \cdot g \cdot sen\theta \cdot \Delta s + T_F = \frac{d \cdot V \cdot v^2}{2} \Rightarrow -\frac{d \cdot V \cdot g \cdot sen\theta \cdot \Delta s}{\Delta t} + \frac{T_F}{\Delta t} = \frac{d \cdot V \cdot v^2}{2 \cdot \Delta t}$$

in which all the terms were divided by Δt. If $Q = \frac{v}{\Delta t}$ (discharge), $P_b = \frac{T_F}{\Delta t}$ (power of the pump) and $d = 1000$ kg m^{-3}, then:

$$-1000.Q.sen\theta.\Delta s + P_b = \frac{d.Q.v^2}{2}.$$

Thus, it follows that

$$P_b = 1000.Q\left(\frac{v^2}{2} + g.sen\theta.\Delta s\right) \quad (3)$$

in which Q is the discharge in m^3.s^{-1}; v is the speed of water exit in m.s^{-1}; $g = 9.81$ m.s^{-2} is the speed of gravity; Δs is the distance of the pumping (m); θ is the slope´s angle in degrees.

According to Putti (2012), for comparison´s sake, the equation of the loss of universal load (Darcy-Weisbach) will be adopted, or rather, maximum speed of water in the pipes will be 2m/s when the diameter of the suction pipes is 48 mm.

Loss of load within the pipes and loss of load caused by the slope are calculated by the dimension of the pipes (Equations 4, 5, 6) (KELLER; BLIESNER, 1990):

$$hf = \frac{8f.L.Q^2}{\pi^2.D^5.g} \quad (4)$$

$$Re = 1.26.10^6\frac{Q}{D} \quad (5)$$

$$f = \frac{1,325}{\left[ln\left(\frac{e}{3.7.D} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad (6)$$

in which L is the length of the pipe (m); Q is the discharge (m³.s^{-1}), D is the diameter (m); g is 9.814 m².s^{-1}, and e is the coefficient of unevenness. The hydraulic pump will be chosen by calculating the following equation (KELLER; BLIESNER, 1990):

$$P_h = \frac{Q.H.\gamma}{n_b.n_m.75} \quad (7)$$

in which P_h is the hydraulic power (cv); Q is discharge (m³.s^{-1}); γ is the water´s specific weight (1000 kgf.m^{-3}); H is the slope plus loss of load (m).

RESULTS AND DISCUSSION

Following methodology proposed for data inserted in the models and theoretical parameters to obtain equations of volume (1) and power (2), the constants were obtained according to the following numerical procedures.

Dimension of the dam´s volume

Since the diameter and depth of the dam are respectively 14 m and 5 m, the Cartesian plane shows depth at point $(0, -5)$, and the lateral points corresponding to the margin are (-7.0) and (7.0) (Figure 4).

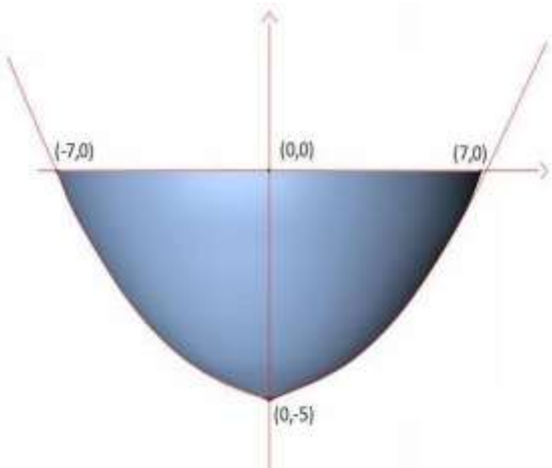

Figure 4 – Cross-section of the dam, with data.

Parameters provide the following:

$$f(x) = \frac{5}{49}x^2 - 5, \qquad 0 \leq x \leq 7, \tag{8}$$

and their inverse:

$$g(y) = \frac{7}{5}\sqrt{5y + 25}, -5 \leq y \leq 0 \tag{9}$$

representing the parabola describing the dam´s cross-section.

The dam´s volume is measured by the equation below:

$$V = \pi \int_{-5}^{0} \left(\frac{7}{5}\sqrt{5y + 25}\right)^2 dy \cong 385m^3 \tag{10}$$

Variation of the dam´s level

Since the amount of irrigation water is 5 mm.day^{-1} in an area of 0.36 hectares, with a 90% efficiency of the irrigation system, the following are required:

$$Q = \frac{ETc \times area}{work\ hours \times efficiency}.$$

Thus, it follows that

$$Q = \frac{0.05 \times 3600}{4 \times 0.9} = 5m^3h^{-1}r^{-1} \quad (11)$$

where ETc is the evapotranspiration of the plant in mm day^{-1}; $area$ in ha; $work\ hours$ in hr; $efficiency$ is decimal. Therefore, variation of water volume in the dam was:

$$\Delta V = V - V_1 = 385 - 20 = 365\ m^3 \quad (12)$$

The integral formula (8) shows its higher limitation equivalent to the level of the margin:

$$365 = \pi \int_{-5}^{b} \left(\frac{7}{5}\sqrt{5y+25}\right)^2 dy$$

Thus,

$$b = -0.072 \quad (13)$$

Therefore, variation in the dam´s level was approximately 7.2 cm (Figure 5).

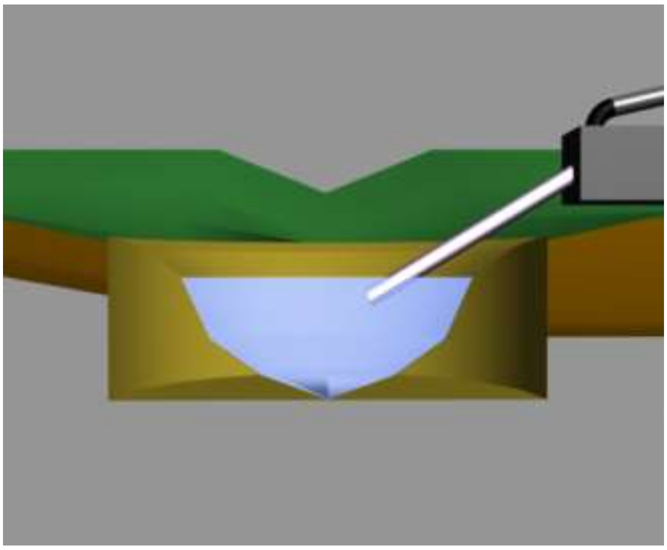

Figure 5 – Variation in the dam´s level.

Consequently, the suction pipes submerged for the capture of water should be at a depth of 7.9 cm. The information is highly relevant to the farmer since the positioning of the lower part of the pipes close to the bottom of the dam may cause the undesired suction of residues and wastes, with serious problems to the pump and engine. Further, the advantages of the engine and pump under water level are enormous since the cavitation of the rotor is avoided.

Dimensioning the required power

Since $V_1 = 20\ m^3$ is required for the expected irrigation (Equation 11) during the period $\Delta t = 4\ h$, the pumping discharge is

$$Q = \frac{V_1}{\Delta t} = \frac{20\ m^3}{4\ h} = \frac{20.000\ L}{4(60)(60)\ s} = 1.39\ \mathrm{L.s^{-1}}$$

Since the distance between the dam and the area to be irrigated is $\Delta s = 380$ m, with a slope $\theta = 21°$, the power expended by the engine could be measured, taking into consideration the speed of water at $V = 1.5$ m.s^{-1}, with a discharge at the exit of $L = 1.38.10^{-3}$ m^3.s^{-1}, by Equation (7):

$$P_b = 1000.1.38.10^{-3}\left(\frac{1.5^2}{2} + (9.81)\text{sen}(21°).380\right) \cong 1845.12 \text{ W} \cong 2.5 \text{ cv}$$

The employment of the Darcy-Weisbach method (KELLER; BLIESNER, 1990) to measure the hydraulic pump first requires the calculation of the load loss. Since the diameter is 48 mm, discharge rate is 5m³ h^{-1}r^{-1}, steel pipes ($e = 0.00015$ m) and length of pipes is 380 m, the load loss at 7.12 m.c.a. and the slope at 21°, will cause a load loss of 79.8 m.c.a.

The pump´s hydraulic power will be the pump´s yield (n_b) at 0.70 and that of the engine (n_m) will be :

$$P_h = \frac{\frac{5}{3600}(86.92)(9.8)}{(0.70)(0.88)(0.735)} = 2.61 \text{ cv}$$

The mathematical models show that the required power was 2.5 cv to pump water from the dam at a distance of 380 m, at a 21° slope, and at a discharge of 5.10^{-3} m.s^{-1}, the discharge needed to pump 20 m^3 of water in four hours. The 385 m^3 dam, represented by a mathematically modeled parabola and given the pumped water volume, would have a variation in level (suppose it is full) of approximately 7.9 cm. The engine would have to be immersed below this borderline level.

CONCLUSIONS

After determining the mathematic models to dimension the water volume in the dam, level variation and power to be developed by the engine under specific irrigation conditions, we may reinforce analytic relationships between power and required water discharge to satisfy the situation.

The performance of the low-power electric engine in water pumping may be assessed by methods contemplated by Physics and by Differential and Integral Calculus. Rates are very close to those calculated by traditional methods. It is possible to show one of the several applications of these theories.

Current analysis shows that electric engines may be associated to the type of service recommended for their use which is highly relevant for the farmer. One must underscore the information on the correct positioning of the lower part of the water pipe to avoid the capture of undesired residues and wastes and even damage to apparatus with grave consequences to the pump and engine. The cavitation of rotors is also avoided.

Future assays will compare the theoretical results obtain in current study (through mathematical and physical principles) and those in the literature. The possibility of providing the required energy by renewable energy sources (wind or photovoltaic) will also be studied, avoiding the use of fossil fuel (diesel) and/or the installation of electric energy network (high installation costs).

SUPPLEMENTARY MATERIAL

A computer simulation of the functioning of the irrigation system may be found at http://www.logicafuzzy.com.br/videos/.

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